# Exercise 4.1

1. Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

Sol. $(x + 1)^2 = 2(x - 3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow x^2 + 7 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

Sol. $x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x + 6 - 2x = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

Sol. $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

$\Rightarrow 3x - 1 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

Sol. $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

Sol. $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$

$\Rightarrow x^2 - 11x + 8 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is a quadratic equation.

(vi) Given, $x^2 + 3x + 1 = (x - 2)^2$

Sol. $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow 7x - 3 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is not a quadratic equation.

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

Sol. $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow x^3 + 2^3 + 3.x.2(x + 2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3.x.2(x - 2)$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 8 + 6x^2 - 12x = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

$\therefore$ It is a quadratic equation.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 $m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Sol. Let the breadth of the rectangular plot = x m

$\therefore$ the length of the plot = (2x + 1) m.

Area of rectangular plot = 528 $m^2$

$\Rightarrow$ length $\times$ breadth = 528 $m^2$

$\Rightarrow (2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

$\therefore$ $2x^2 + x - 528 = 0$ is the required representation of the problem mathematically.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

Sol. Let the two consecutive positives integers are x and x + 1.

A. t. q. $x \times (x + 1) = 306$

$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

$\therefore$ $x^2 + x - 306 = 0$ is the required representation of the problem mathematically.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Sol. Let the age of Rohan's = x years

$\therefore$ age of his mother = x + 26 years

After 3 years,

Age of Rohan = x + 3 years

And age of his mother = x + 26 + 3

= x + 29 years

A. t. q. The product of their ages after 3 years = 360

$\Rightarrow (x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

$\therefore$ $x^2 + 32x - 273 = 0$ is the required representation of the problem mathematically.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol. Sol. Let the speed of train = x km/h

Time taken to travel 480 km = $\frac{480}{x}$ km/hr

New speed of train = (x – 8) km/h

∴ Time taken to travel 480 km = $\frac{480}{x-8}$ km/hr

A. t. q.

$\Rightarrow \frac{480}{x} + 3 = \frac{480}{x-8}$

$\Rightarrow 3 = \frac{480}{x-8} - \frac{480}{x}$

$\Rightarrow 3 = \frac{480x-480\,(x-8)}{x(x-8)}$

$\Rightarrow 3\,x(x-8) = 480x - 480x + 3840$

$\Rightarrow 3x^2 - 24\,x - 3840 = 0$

$\Rightarrow x^2 - 8\,x - 1280 = 0$

∴ $x^2 - 8\,x - 1280 = 0$ is the required representation of the problem mathematically.

## Exercise 4.2

1. Find the roots of the following quadratic equations by factorization:

(i) $x^2 - 3x - 10 = 0$

Sol. $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow (x - 5) = 0$ or $(x + 2) = 0$

$\Rightarrow x = 5$ or $x = -2$

∴ roots of the quadratic equation are $x = 5, -2$ Ans.

(ii) $2x^2 + x - 6 = 0$

Sol. $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

⇒ $2x(x + 2) - 3(x + 2) = 0$

⇒ $(x + 2)(2x - 3) = 0$

⇒ $x + 2 = 0$ or $2x - 3 = 0$

⇒ $x = -2$ or $x = \frac{3}{2}$

∴ roots of the quadratic equation are $x = -2, \frac{3}{2}$ Ans.

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Sol. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

⇒ $\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

⇒ $x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

⇒ $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

⇒ $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

⇒ $x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

∴ roots of the quadratic equation are $x = \frac{-5}{\sqrt{2}}, -\sqrt{2}$ Ans.

(iv) $2x^2 - x + \frac{1}{8} = 0$

Sol. $2x^2 - x + \frac{1}{8} = 0$

⇒ $16x^2 - 8x + 1 = 0$

⇒ $16x^2 - 4x - 4x + 1 = 0$

⇒ $4x(4x - 1) - 1(4x - 1) = 0$

⇒ $(4x - 1)(4x - 1) = 0$

⇒ $(4x - 1) = 0$ or $(4x - 1) = 0$

⇒ $x = \frac{1}{4}$ or $x = \frac{1}{4}$

∴ roots of the quadratic equation are $x = \frac{1}{4}, \frac{1}{4}$ Ans.

(v) $100x^2 - 20x + 1 = 0$

Sol. $100x^2 - 20x + 1 = 0$

⇒ $100x^2 - 10x - 10x + 1 = 0$

⇒ $10x(10x - 1) - 1(10x - 1) = 0$

⇒ $(10x - 1)(10x - 1) = 0$

⇒ $(10x - 1) = 0$ or $(10x - 1) = 0$

⇒ $x = \frac{1}{10}$ or $x = \frac{1}{10}$

∴ roots of the quadratic equation are $x = \frac{1}{10}, \frac{1}{10}$ Ans.

2. Solve the problems given in Example 1.

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Sol. Let the number of marbles John have = x.
Therefore, number of marbles Jivanti have = 45 – x
After losing 5 marbles each,
Number of marbles John have = x – 5
Number of marbles Jivanti have = 45 – x – 5
= 40 – x
A. t. q. now the product of their marbles is 124.
$\therefore (x - 5)(40 - x) = 124$
$\Rightarrow 40x - x^2 - 200 + 5x - 124 = 0$
$\Rightarrow -x^2 + 45x - 324 = 0$
$\Rightarrow x^2 - 45x + 324 = 0$
$\Rightarrow x^2 - 36x - 9x + 324 = 0$
$\Rightarrow x(x - 36) - 9(x - 36) = 0$
$\Rightarrow (x - 36)(x - 9) = 0$
$\Rightarrow x - 36 = 0$ or $x - 9 = 0$
$\Rightarrow x = 36$ or $x = 9$
Therefore,
If, John's marbles = 36,
Then, Jivanti's marbles = 45 – 36 = 9
Or, if John's marbles = 9,
Then, Jivanti's marbles = 45 – 9 = 36 Ans.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ` 750. We would like to find out the number of toys produced on that day.

Sol. Let the number of toys produced in a day = x.
Therefore, cost of production of each toy that day = Rs (55 – x)
A. t. q. total cost of production of the toys = Rs 750
$\Rightarrow x(55 - x) = 750$
$\Rightarrow 55x - x^2 - 750 = 0$
$\Rightarrow -x^2 + 55x - 750 = 0$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

$\Rightarrow x - 25 = 0$ or $x - 30 = 0$

$\Rightarrow x = 25$ or $x = 30$

∴ The number of toys produced in a day, will be either 25 or 30. Ans.

**3. Find two numbers whose sum is 27 and product is 182.**

Sol. Let the first number = $x$

∴ the second number = $27 - x$.

A. t. q. the product of two numbers = 182

$\Rightarrow x(27 - x) = 182$

$\Rightarrow x^2 - 27x - 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

$\Rightarrow x - 13 = 0$ or $x - 14 = 0$

$\Rightarrow x = 13$ or $x = 14$

Therefore, if first number = 13, then second number = 27 – 13 = 14

Or, if first number = 14, then second number = 27 – 14 = 13

∴ The numbers are 13 and 14. Ans.

**4. Find two consecutive positive integers, sum of whose squares is 365.**

Sol. Let the two consecutive positive integers are $x$ and $x + 1$.

A. t. q. sum of squares of the numbers = 365

$\Rightarrow x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x - 365 = 0$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$\Rightarrow (x + 14)(x - 13) = 0$

$\Rightarrow x + 14 = 0$ or $x - 13 = 0$,

$\Rightarrow x = -14$ or $x = 13$

∴ $x = 13$ [∵ $x$ is a positive integer]

∴ The two consecutive positive integers are 13 and 13+1 = 14. Ans.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol. Let the base of the right triangle = x cm.

∴ the altitude of right triangle = (x – 7) cm

In a right triangle, we know

$Base^2 + Altitude^2 = Hypotenuse^2$ [By Pythagoras theorem]

⇒ $x^2 + (x - 7)^2 = 13^2$

⇒ $x^2 + x^2 - 14x + 49 = 169$

⇒ $2x^2 - 14x + 49 - 169 = 0$

⇒ $2x^2 - 14x - 120 = 0$

⇒ $x^2 - 7x - 60 = 0$

⇒ $x^2 - 12x + 5x - 60 = 0$

⇒ $x (x - 12) + 5 (x - 12) = 0$

⇒ $(x - 12) (x + 5) = 0$

⇒ $x - 12 = 0$ or $x + 5 = 0$,

⇒ $x = 12$ or $x = -5$

∴ $x = 12$ [∵ sides cannot be negative]

∴ The base = 12cm and the altitude = 12 – 7 cm = 5 cm. Ans.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Sol. Let the number of articles produced = x.

∴ the cost of production of each article = Rs (2x + 3)

A. t. q. total cost of production = Rs. 90

∴ $x (2x + 3) = 90$

⇒ $2x^2 + 3x - 90 = 0$

⇒ $2x^2 + 15x - 12x - 90 = 0$

⇒ $x (2x + 15) - 6 (2x + 15) = 0$

⇒ $(2x + 15) (x - 6) = 0$

⇒ $2x + 15 = 0$ or $x - 6 = 0$

⇒ $x = \frac{-15}{2}$ or $x = 6$

∴ $x = 6$ [∵ the number of articles produced can only be a positive integer]

∴ The number of articles produced = 6

Cost of each article = 2 × 6 + 3 = Rs 15. Ans.

## Exercise 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;

(i) $2x^2 - 3x + 5 = 0$

Sol. $2x^2 - 3x + 5 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we have,

$a = 2$, $b = -3$ and $c = 5$

$\therefore$ Discriminant $= b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31 < 0$

$\therefore$ equation has not real roots. Ans.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Sol. $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we have,

$a = 3$, $b = -4\sqrt{3}$ and $c = 4$

$\therefore$ Discriminant $= b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48$

$= 0$

Roots are real and equal.

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4\sqrt{3}) \pm \sqrt{0}}{2\,.\,3}$

$= \frac{-4\sqrt{3} \pm 0}{6}$

$= \frac{-4\sqrt{3} + 0}{6}$ or $\frac{-4\sqrt{3} - 0}{6}$

$= \frac{-2\sqrt{3}}{3}$ or $\frac{-2\sqrt{3}}{3}$

Therefore, the roots are $x = \frac{-2\sqrt{3}}{3}, \frac{-2\sqrt{3}}{3}$ Ans.

(iii) $2x^2 - 6x + 3 = 0$

Sol. $2x^2 - 6x + 3 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we have a = 2, b = -6, c = 3

∴ Discriminant = $b^2 - 4ac$

= $(-6)^2 - 4(2)(3)$

= 36 – 24

= 12 > 0

∴ Roots are real and distinct.

∴ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-6) \pm \sqrt{12}}{2\,.\,2}$

$= \frac{6 \pm 2\sqrt{3}}{4}$

$= \frac{2(3 \pm \sqrt{3})}{4}$

$= \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

∴ the roots for the given equation are $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$ Ans.

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

Sol. $2x^2 + kx + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have,

a = 2, b = k and c = 3

Since the roots are real and equal

⇒ $b^2 - 4ac = 0$

⇒ $k^2 - 4.2.3 = 0$

⇒ $k^2 = 24$

⇒ $k = \pm\sqrt{24}$

∴ $k = \pm 2\sqrt{6}$ Ans.

(ii) kx (x – 2) + 6 = 0

Sol. kx (x – 2) + 6 = 0

or $kx^2 - 2kx + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have,

a = k, b = -2k and c = 6

Since the roots are real and equal

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4(k)(6) = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k - 6) = 0$

$\Rightarrow 4k = 0$ or $k = 6 = 0$

$\Rightarrow k = 0$ or $k = 6$

$\therefore k = 6\ [\because k \neq 0]$

$\therefore$ value of k is 6 Ans.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 $m^2$? If so, find its length and breadth.

Sol. Let the breadth of mango grove = x m

$\therefore$ Length of mango grove = 2x m

Area of mango grove = 800 $m^2$

$\Rightarrow$ length $\times$ breadth = 800 $m^2$

$\Rightarrow 2x \times x = 800$

$\Rightarrow 2x^2 = 800$

$\Rightarrow x^2 = 400$

$\Rightarrow x^2 - 400 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1, b = 0, c = 400$

$\therefore$ Discriminant $= b^2 - 4ac$

$= (0)^2 - 4 \times (1) \times (-400)$

$= 1600 > 0$

$\therefore$ Roots are real and distinct.

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-0 \pm \sqrt{1600}}{2\,.\,1}$

$= \frac{-0 \pm 40}{2}$

$= 20, -20$

$\therefore x = 20$ [$\because$ length never be negative]

$\therefore$ breadth of mango grove = 20 m

And length of mango grove = 2 × 20 = 40 m Ans.

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol. Let the age of one friend = x years.

∴ the age of the other friend = (20 – x) years.

Four years ago, age of first friend = (x – 4) years

And the age of second friend = (20 – x – 4) = (16 – x) years

A. t. q. the product of their ages = 48 years

⇒ $(x - 4)(16 - x) = 48$

⇒ $16x - x^2 - 64 + 4x = 48$

⇒ $-x^2 + 20x - 64 - 48 = 0$

⇒ $-x^2 + 20x - 112 = 0$

⇒ $x^2 - 20x + 112 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we have,

$a = 1$, $b = -20$ and $c = 112$

∴ Discriminant = $b^2 - 4ac$

$= (-20)^2 - 4 \times 112$

$= 400 - 448$

$= -48 < 0$

∴ equation has not real roots.

Hence this situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 $m^2$? If so, find its length and breadth.

Sol. Let the length of the park = x m

Perimeter of the rectangular park = 80

⇒ 2 (length + breadth) = 80

⇒ x + breadth = $\frac{80}{2}$

⇒ breadth = 40 - x

Area of rectangular park 400 $m^2$

⇒ length × breadth = 400 $m^2$

⇒ $x \times (40 - x) = 400$

⇒ $40x - x^2 - 400 = 0$

⇒ $x^2 - 40x + 400 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -40$, $c = 400$

∴ Discriminant = $b^2 - 4ac$

$= (-40)^2 - 4 \times (1) \times (400)$

$= 1600 - 1600$

$= 0$

∴ equation has real and equal roots.

Hence, the situation is possible.

∴ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-40) \pm \sqrt{0}}{2 \, . \, 1}$

$= \frac{40 \pm 0}{2}$

$= 20, 20$

∴ length of rectangular park = 20 m

And breadth of the park, b = 40 – 20 = 20 m Ans.